Calculus III- Worksheet #8

1. Find the equation of the tangent plane and normal line of the surface $xy + yz + zx = 5$ at (1,2,1).

2. Find the local minimum, local maximum and saddle point(s) of the function $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

3. Find the absolute maximum and absolute minimum of the function $f(x, y) = x^4 + y^4 - 4xy + 2$ on $D = \{(x, y)|0 \leq x \leq 4, 0 \leq y \leq 2\}$.

4. Find the absolute maximum and absolute minimum of the function $f(x, y) = 2x^3 + y^4$ on $D = \{(x, y)|x^2 + y^2 \leq 1\}$.

5. Using Lagrange multipliers find the maximum and minimum of the function $f(x, y) = x^4 + y^4 + z^4$ with the constraint $x^2 + y^2 + z^2 = 1$.

6. Using Lagrange multipliers find the three positive numbers whose sum is 100 and whose product is a maximum.

7. Two surfaces are called orthogonal at a point of intersection if their normal lines are perpendicular at that point. Show that the surfaces with equations $F(x, y, z) = 0$ and $G(x, y, z) = 0$ are orthogonal at a point **P** where $\nabla F \neq 0 \ and \ \nabla G \neq 0$ if and only if
$$F_x G_x + F_y G_y + F_z G_z = 0 \ at \ \boldsymbol{P}.$$