

Calculus III: Worksheet #1 – Section 12.2-12.3

Directions: Solve no more than one solution per page. Show all your work and the steps.

Q1. Find $\mathbf{a} + \mathbf{b}$, $2\mathbf{a} + 3\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a} - \mathbf{b}|$ of $\mathbf{a} = \langle 5, -12 \rangle$ and $\mathbf{b} = \langle -3, -6 \rangle$.

Q2. Find a unit vector of $8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

Q3. Find the angle between the vector $\mathbf{i} + \sqrt{3}\mathbf{j}$ and positive direction of x-axis.

Q4. Problem 37 in Section 12.2.

Q5. Find the angle between the vectors $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$.

Q6. A. Determine whether the given vectors are orthogonal, parallel or neither

i. $\mathbf{a} = \langle -5, 3, 7 \rangle$, $\mathbf{b} = \langle 6, -8, 2 \rangle$

ii. $\mathbf{a} = \langle -1, 2, 5 \rangle$, $\mathbf{b} = \langle 3, 4, -1 \rangle$

B. Find the direction cosines of $\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$.

Q7. Find the scalar and vector projections of $\mathbf{b} = \langle 2, -1, 4 \rangle$ onto $\mathbf{a} = \langle 0, 1, \frac{1}{2} \rangle$

Q8. Find the work done by a Force $\mathbf{F} = \langle 8, -6, 9 \rangle$ that moves an object from the point $(0, 10, 8)$ to the point $(6, 12, 20)$ along a straight line. Distance is measured in meters and the force in newtons.

Q9. Show that $|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2|\mathbf{a}|^2 + 2|\mathbf{b}|^2$ for any vector \mathbf{a} and \mathbf{b} .

Q10. If $\mathbf{r} = \langle x, y, z \rangle$, $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, show that the vector equation $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$ represents a sphere, find its center and radius.