H.W. 13

Using laplace Transforms solve:

1. $y(t) = \cos t + \int_0^t e^{-z}\, y(t-z)\, dz$.

2. $y^{'}(t) + 6y + 9\int_0^t y(z)\, dz = 1$ with initial conditions $y(0) = 0$.

3. $y^{''} - 6y^{'} + 9y = t^2 e^{3t}$ with initial conditions $y(0) = 2$, $y^{'}(0) = 6$

4. $y^{''} + 4y = f(t)$ where

$$f(t) = \begin{cases} 0; & 0 \le t < \pi \\ 1; & \pi \le t < 2\pi \\ 0; & t \ge 2\pi \end{cases}$$

with initial conditions $y(0) = 1$, $y^{'}(0) = 0$.

5. $x^{'} = 6x + y$
$y^{'} = 4x + 3y$
with initial conditions $x(0) = 2$, $y(0) = 7$.

6. $x^{''} + 16x = f(t)$ where

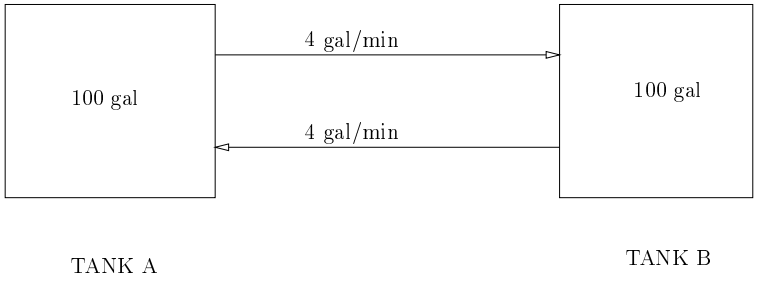
$$f(t) = \begin{cases} \cos 4t; & 0 \le t < \pi \\ 0; & t \ge \pi \end{cases}$$

with initial conditions $x(0) = 0$, $x^{'}(0) = 1$.

7. Solve $x^{'} = x + 3y$, $y^{'} = 3x + y$ with initial conditions $x(0) = 1$, $y(0) = 0$.

8. Solve $x^{'} = 3x - 3y$, $y^{'} = 6x - 3y$ with initial conditions $x(0) = 4$, $y(0) = 3$.

9. A closed two tank system has the configuration given below. The initial data for each tank is: Tank A contains 8 lb of chemical X dissolved in 100 gal of fresh water. Tank B contains 4 lb of chemical X dissolved in 100 gal of fresh water.




Assuming that the mixing devices maintain homogeneity,

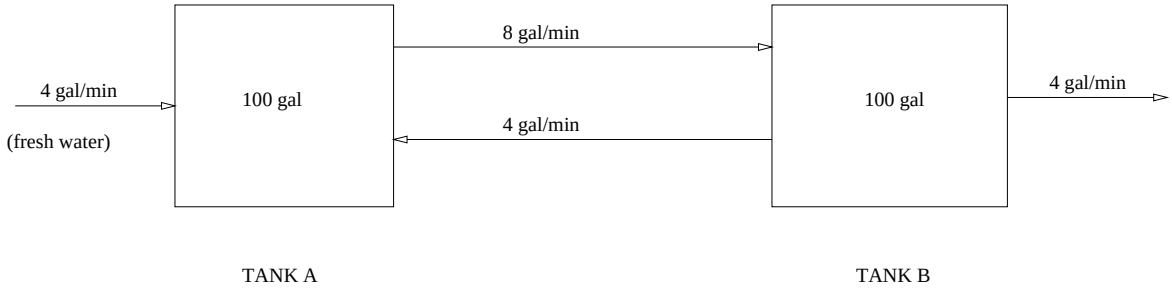
(a). Find the amounts $x(t)$, $y(t)$ of chemical X in Tanks A and B at time $t$.

(b). When will the amount of chemical in Tank B be 6 lb?

10. A two-tank system has the configuration given below. At time $t = 0$ Tank A contains 100 gal of fresh water, whereas Tank B contains 100 gal of brine consisting of 10 lb of dissolved salt. Assuming that the mixture devices maintain homogeneity,

    (a). Find the amount of salt $x(t)$ in Tank A and the amount of salt $y(t)$ in Tank B at time $t$.

    (b). Also show that after a long time both tanks will essentially contain fresh water.




11. Solve $y'' + 3y' + 2y = f(t)$, $y(0) = 0$, $y'(0) = 0$ where

$$f(t) = \begin{cases} 1; & t < 2 \\ -1; & 2 \le t < 4 \\ 0; & t \ge 4. \end{cases}$$

Also find $y(3)$.

12. Solve $y'' + y = g(t)$, $y(0) = 0$, $y'(0) = 0$ where

$$g(t) = \begin{cases} 0; & t < 2\pi \\ \sin t; & t \geq 2\pi \end{cases}$$

13. Solve $y'' + 5y' + 4y = 0$; $y(0) = 1$, $y'(0) = 0$.

14. Solve $y'' - 6y' + 9y = t$; $y(0) = 0$, $y'(0) = 1$.

15. Solve $2y''' + 3y'' - 3y' - 2y = e^{-t}$; $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$.

16. Solve $f(t) + \int_0^t (t - \tau) f(\tau) d\tau = t$.

17. Solve $f(t) = 1 + t - \frac{8}{3} \int_0^t (\tau - t)^3 f(\tau) d\tau$.

18. Solve $f(t) + \int_0^t f(\tau) d\tau = 1$.