H.W. 11 Solution

Q1)

$$y'' + y = tanx$$

$$y'' + y = 0(m^2 + 1 = 0, m = \pm i) \Longrightarrow y_c = C_1 cosx + C_2 sinx$$

$$\text{Let } y_p = Acosx + Bsinx \ \text{ such that}$$

$$A'cosx + B'sinx = 0$$

$$\overset{DE}{\Longrightarrow} A'(-sinx) + B'(cosx) = tanx$$

$$\Longrightarrow A' = -\frac{sin^2x}{cosx} = cosx - secx$$

$$\Longrightarrow A = sinx - ln|secx + tanx|$$

$$\text{Also } B' = sinx \Longrightarrow B = -cosx$$

$$\Longrightarrow y_p = \{sinx - ln|secx + tanx|\}cosx + (-cosx)sinx$$

$$= -cosxln|secx + tanx|$$

$$\Longrightarrow \text{G. S. } y = y_c + y_p = C_1 cosx + C_2 sinx - cosxln|secx + tanx|$$

Q2)

$$(x^2 + 1)y'' - 2xy' + 2y = 6(x^2 + 1)^2$$

$$y_1 = x$$

$$y_2 = x^2 - 1$$

$$\text{Let } y_p = Ax + B(x^2 - 1) \ \text{ such that}$$

$$A'x + B'(x^2 - 1) = 0$$

$$\overset{DE}{\Longrightarrow} A' \cdot 1 + B'(2x) = 6(x^2 + 1)$$

$$\text{Also } B'(x^2 + 1) = 6x(x^2 + 1)$$

$$\Longrightarrow B' = 6x \Longrightarrow B = 3x^2$$

$$\Longrightarrow A' = -6(x^2 - 1) \Longrightarrow A = 6x - 2x^3$$

$$\Longrightarrow y_p = (6x - 2x^3)x + 3x^2(x^2 - 1) = 3x^2 + x^4$$

$$\Longrightarrow \text{G. S. } y = y_c + y_p = C_1 x + C_2(x^2 - 1) + (3x^2 + x^4)$$

Q3)

$$y'' + y = tanxsecx$$

$$y'' + y = 0(m^2 + 1 = 0, m = \pm i) \Longrightarrow y_c = C_1 cosx + C_2 sinx$$

$$\text{Let } y_p = Acosx + Bsinx \ \text{ such that}$$

$$A'cosx + B'sinx = 0$$

$$\stackrel{DE}{\Longrightarrow} A'(-sinx) + B'(cosx) = tanxsecx$$

$$\Longrightarrow B' = tanx$$

$$\Longrightarrow B = -ln|cosx|$$

$$\Longrightarrow A' = -tan^2 x = 1 - sec^2 x$$

$$\Longrightarrow A = x - tanx$$

$$\Longrightarrow y_p = (x - tanx)cosx - (ln|cosx|)sinx$$

$$= xcosx - sinx - sinxln|cosx|$$

$$\Longrightarrow \text{G. S. } y = y_c + y_p = C_1 cosx + C_2 sinx + xcosx - sinxln|cosx| - sinx$$

Q4)

$$y'' + 6y' + 9y = \frac{e^{-3x}}{x^3}$$

$$y'' + 6y' + 9y = 0$$

$$m^2 + 6m + 9 = 0$$

$$(m + 3)^2 = 0$$

$$m = -3, -3$$

$$y_c = C_1 e^{-3x} + C_2 x e^{-3x}$$

$$\text{Let } y_p = Ae^{-3x} + B(xe^{-3x}) \ \text{ such that}$$

$$A'e^{-3x} + B'(xe^{-3x}) = 0$$

$$\overset{DE}{\Longrightarrow} A'(-3e^{-3x}) + B'(e^{-3x} - 3xe^{-3x}) = \frac{e^{-3x}}{x^3}$$

$$\text{Also } B'e^{-3x} = \frac{e^{-3x}}{x^3}$$

$$\Longrightarrow B' = x^{-3} \Longrightarrow B = \frac{x^{-2}}{-2}$$

$$\Longrightarrow A' = -x^{-2} \Longrightarrow A = x^{-1}$$

$$\Longrightarrow y_p = x^{-1}e^{-3x} + \frac{x^{-2}}{-2}xe^{-3x} = \frac{1}{2}x^{-1}e^{-3x}$$

$$\Longrightarrow \text{G. S. } y = y_c + y_p = C_1e^{-3x} + C_2xe^{-3x} + \frac{1}{2}x^{-1}e^{-3x}$$

Q5)

$$y'' - 2y' + y = xe^x lnx; \quad x > 0$$

$$y'' - 2y' + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$y_c = C_1e^x + C_2xe^x$$

$$\text{Let } y_p = Ae^x + B(xe^x) \text{ such that}$$

$$A'e^x + B'(xe^x) = 0$$

$$\overset{DE}{\Longrightarrow} A'(e^x) + B'(xe^x + e^x) = xe^x lnx$$

$$\Longrightarrow B'e^x = xe^x lnx$$

$$\Longrightarrow B' = xlnx$$

$$\Longrightarrow B = \int xlnxdx$$

$$= \int lnx(\frac{x^2}{2})'dx$$

$$= (\frac{x^2}{2})lnx - \int \frac{x^2}{2}\frac{1}{x}dx$$

$$= \frac{x^2}{2} lnx - \frac{x^2}{4}$$

$$\Longrightarrow A' = -x^2 lnx$$

$$\Longrightarrow A = -\int (lnx)(\frac{x^3}{3})' dx$$

$$= -(\frac{x^3}{3}) lnx + \int \frac{x^3}{3} \frac{1}{x} dx$$

$$= -\frac{x^3}{3} lnx + \frac{x^3}{9}$$

$$\Longrightarrow y_p = \{-\frac{x^3}{3} lnx + \frac{x^3}{9}\} e^x + \{\frac{x^2}{2} lnx - \frac{x^2}{4}\} x e^x$$

$$= \{\frac{1}{6} x^3 lnx - \frac{5}{36} x^3\} e^x$$

$$\Longrightarrow \text{G. S. } y = y_c + y_p = C_1 e^x + C_2 x e^x + \{\frac{1}{6} x^3 lnx - \frac{5}{36} x^3\} e^x$$

Q6)

$$x^2 y'' - 6xy' + 10y = 3x^4 + 6x^3$$

$$y_1 = x^2$$

$$y_2 = x^5$$

$$\text{Let } y_p = Ax^2 + Bx^5) \text{ such that}$$

$$A'x^2 + B'x^5 = 0$$

$$\overset{DE}{\Longrightarrow} A'(2x) + B'(5x^4) = 3x^2 + 6x$$

$$\Longrightarrow (-B'x^3)(2x) + B'(5x^4) = 3x^2 + 6x$$

$$\Longrightarrow 3x^4 B' = 3x^2 + 6x$$

$$\Longrightarrow B' = \frac{1}{x^2} + \frac{2}{x^3}$$

$$\Longrightarrow B = \frac{x^{-1}}{-1} + 2\frac{x^{-2}}{-2}$$

$$A' = -(x+2) \Longrightarrow A = -\frac{x^2}{2} - 2x$$

$$\Longrightarrow y_p = (-\frac{x^2}{2} - 2x)x^2 + (\frac{x^{-1}}{-1} - 2x^{-2})x^5$$

$$= -\frac{x^4}{2} - 2x^3 - x^4 - x^3$$

$$= -\frac{3}{2}x^4 - 3x^3$$

$$\Longrightarrow \text{G. S. } y = y_c + y_p = C_1x^2 + C_2x^5 - \frac{3}{2}x^4 - 3x^3$$

Q7)

$$y'' + y = tan^3x$$

$$y'' + y = 0(m^2 + 1 = 0, m = \pm i) \Longrightarrow y_c = C_1cosx + C_2sinx$$

$$\text{Let } y_p = Acosx + Bsinx \text{ such that}$$

$$A'cosx + B'sinx = 0$$

$$\overset{DE}{\Longrightarrow} A'(-sinx) + B'(cosx) = tan^3x$$

$$\Longrightarrow B' = \frac{sin^3x}{cos^2x}$$

$$\Longrightarrow B = \int \frac{sin^2x}{cos^2x} sinxdx$$

$$= \int \frac{1 - cos^2x}{cos^2x}(sinxdx)$$

$$\overset{v=cosx}{\Longrightarrow} = \int \frac{1 - v^2}{v^2}(-dv)$$

$$= \int (1 - v^{-2})dv$$

$$= v + \frac{1}{v} = cosx + \frac{1}{cosx}$$

$$\Longrightarrow A' = -\frac{sin^4x}{cos^3x}$$

$$\Longrightarrow A = \int -\frac{sin^4x}{cos^3x}dx$$

$$= \frac{1}{2}\int sin^3x(\frac{1}{cos^2x})'dx$$

$$= \frac{1}{2}\frac{sin^3x}{cos^2x} - \frac{1}{2}\int \frac{1}{cos^2x}3sin^2xcosxdx$$

$$= \frac{1}{2}\frac{sin^3x}{cos^2x} - \frac{3}{2}\int \frac{1 - cos^2x}{cosx}dx$$

$$= \frac{1}{2}\frac{sin^3x}{cos^2x} - \frac{3}{2}\int [secx - cosx]dx$$

$$= \frac{1}{2}\frac{sin^3x}{cos^2x} - \frac{3}{2}ln|secx + tanx| + \frac{3}{2}sinx$$

$$\Longrightarrow y_p = \frac{1}{2}\frac{sin^3x}{cosx} - \frac{3}{2}cosxln|secx + tanx| + \frac{3}{2}cosxsinx + cosxsinx + \frac{sinx}{cosx}$$

$$\Longrightarrow \text{G. S. } y = y_c + y_p = C_1cosx + C_2sinx + \frac{1}{2}\frac{sin^3x}{cosx} - \frac{3}{2}cosxln|secx + tanx| + \frac{5}{2}cosxsinx + \frac{sinx}{cosx}$$

Q8)

$$y'' + y = secx$$

$$y'' + y = 0(m^2 + 1 = 0, m = \pm i) \Longrightarrow y_c = C_1cosx + C_2sinx$$

$$\text{Let } y_p = Acosx + Bsinx \ \text{ such that}$$

$$A'cosx + B'sinx = 0$$

$$\overset{DE}{\Longrightarrow} A'(-sinx) + B'(cosx) = secx$$

$$\Longrightarrow B' = 1 \Longrightarrow B = x$$

$$\Longrightarrow A' = -\frac{sinx}{cosx}$$

$$\Longrightarrow A = ln|cosx|$$

$$\Longrightarrow y_p = cosxln|cosx| + xsinx$$

$$\Longrightarrow \text{G. S. } y = y_c + y_p = C_1cosx + C_2sinx + cosxln|cosx| + xsinx$$