

H.W. 7 Solution

Q1)

$$y'' + 9y = 0$$

$$\text{CH-EQ: } m^2 + 9 = 0$$

$$\implies m = \pm 3i$$

$$\implies \text{G. S. } y = C_1 \cos(3x) + C_2 \sin(3x)$$

where C_1 & C_2 are arbitrary constants.

Q2)

$$y'' + y = 0; \quad y(0) = 1, \quad y'(0) = 0$$

$$\text{CH-EQ: } m^2 + 1 = 0$$

$$\implies m = \pm i$$

$$\implies \text{G. S. } y = C_1 \cos x + C_2 \sin x$$

$$y(0) = 1 \implies C_1 = 1$$

$$y'(0) = 0 \implies C_2 = 0$$

$$\text{P. S. } \boxed{y = \cos x}$$

Q3)

$$y^{(4)} + 8y^{(2)} + 16y = 0$$

$$\text{CH-EQ: } m^4 + 8m^2 + 16 = 0$$

$$\implies (m^2 + 4)^2 = 0$$

$$\implies m = \pm 2i, \pm 2i$$

$$\implies \text{G. S. } y = C_1 \cos(2x) + C_2 \sin(2x) + C_3 x \cos(2x) + C_4 x \sin(2x)$$

where C_1, C_2, C_3 & C_4 are arbitrary constants.

Q4)

$$y^{(3)} - 3y^{(2)} + 9y^{(1)} + 13y = 0$$

$$\text{CH-EQ: } f(m) = m^3 - 3m^2 + 9m + 13 = 0$$

$$\text{Note } f(-1) = -1 - 3 - 9 + 13 = 0$$

$$\implies (m + 1)(m^2 - 4m + 13) = 0$$

$$\implies m = -1, \frac{4 \pm \sqrt{16 - 4 \cdot 13}}{2}$$

$$= -1, 2 \pm 3i$$

$$\implies \text{G. S. } y = C_1 e^{-x} + C_2 e^{2x} \cos 3x + C_3 e^{2x} \sin 3x$$

where C_1, C_2 & C_3 are arbitrary constants.

Q5)

$$y^{(4)} + 5y^{(2)} - 36y = 0$$

$$\text{CH-EQ: } m^4 + 5m^2 - 36 = 0$$

$$\implies (m^2 + 9)(m^2 - 4) = 0$$

$$\implies m = \pm 3i, 2, -2$$

$$\implies \text{G. S. } y = C_1 \cos 3x + C_2 \sin 3x + C_3 e^{2x} + C_4 e^{-2x}$$

where C_1, C_2, C_3 & C_4 are arbitrary constants.