

H.W. 6 Solution

Q1)

$$y'' + y' - 6y = 0$$

$$\text{CH-EQ: } m^2 + m - 6 = 0$$

$$\implies (m - 2)(m + 3) = 0$$

$$\implies m = 2, -3$$

$$\implies \text{G. S. } y = C_1 e^{2x} + C_2 e^{-3x}$$

where C_1 & C_2 are arbitrary constants.

Q2)

$$y''' - y'' - 4y' + 4y = 0$$

$$\text{CH-EQ: } m^3 - m^2 - 4m + 4 = 0$$

$$\implies m^2(m - 1) - 4(m - 1) = 0$$

$$\implies (m - 1)(m^2 - 4) = 0$$

$$\implies (m - 1)(m - 2)(m + 2) = 0$$

$$\implies m = 1, 2, -2$$

$$\implies \text{G. S. } y = C_1 e^x + C_2 e^{2x} + C_3 e^{-2x}$$

where C_1, C_2 & C_3 are arbitrary constants.

Q3)

$$y''' + 2y'' - 5y' - 6y = 0$$

$$\text{CH-EQ: } f(m) = m^3 + 2m^2 - 5m - 6 = 0$$

$$\text{Note } f(2) = 8 + 8 - 10 - 6 = 0$$

$$\implies (m - 2)(m^2 + 4m + 3) = 0$$

$$\implies (m - 2)(m + 3)(m + 1) = 0$$

$$\implies m = 2, -3, -1$$

$$\implies \text{G. S. } y = C_1 e^{2x} + C_2 e^{-3x} + C_3 e^{-x}$$

where C_1, C_2 & C_3 are arbitrary constants.

Q4)

$$y'' - 2y' - 3y = 0; \quad y(0) = 0, \quad y'(0) = -4$$

$$\text{CH-EQ: } m^2 - 2m - 3 = 0$$

$$\implies (m - 3)(m + 1) = 0$$

$$\implies m = 3, -1$$

$$\implies \text{G. S. } y = C_1 e^{3x} + C_2 e^{-x}$$

$$y(0) = 0 \implies C_1 + C_2 = 0$$

$$y'(0) = -4 \implies 3C_1 - C_2 = -4$$

$$\implies 4C_1 = -4 \implies C_1 = -1 \implies C_2 = 1$$

$$\text{P. S. } \boxed{y = -e^{3x} + e^{-x}}$$

Q5)

$$y'' - 2y' + y = 0$$

$$\text{CH-EQ: } m^2 - 2m + 1 = 0$$

$$\implies (m - 1)^2 = 0$$

$$\implies m = 1, 1$$

$$\implies \text{G. S. } y = C_1 e^x + C_2 x e^x$$

where C_1 & C_2 are arbitrary constants.

Q6)

$$y''' - 3y'' + 3y' - y = 0$$

$$\text{CH-EQ: } f(m) = m^3 - 3m^2 + 3m - 1 = 0$$

$$\text{Note } f(1) = 1 - 3 + 3 - 1 = 0$$

$$\implies (m - 1)(m^2 - 2m + 1) = 0$$

$$\implies (m-1)(m-1)(m-1) = 0$$

$$\implies m = 1, 1, 1$$

$$\implies \text{G. S. } y = C_1 e^x + C_2 x e^x + C_3 x^2 e^x$$

where C_1, C_2 & C_3 are arbitrary constants.

Q7)

$$y'''' + 6y''' + 5y'' - 24y' - 36y = 0$$

$$\text{CH-EQ: } m^4 + 6m^3 + 5m^2 - 24m - 36 = 0$$

$$\implies (m^4 + 5m^2 - 36) + (6m^3 - 24m)$$

$$\implies (m^2 + 9)(m^2 - 4) + 6m(m^2 - 4)$$

$$\implies (m^2 - 4)(m^2 + 9 + 6m) = 0$$

$$\implies (m-2)(m+2)(m+3)(m+3) = 0$$

$$\implies m = 2, -2, -3, -3$$

$$\implies \text{G. S. } y = C_1 e^{2x} + C_2 e^{-2x} + C_3 e^{-3x} + C_4 x e^{-3x}$$

where C_1, C_2, C_3 & C_4 are arbitrary constants.