H.W. 5 Solution

Q1)

$$\underbrace{(2xsiny + y^3e^x)}_{M} + \underbrace{(x^2cosy + 3y^2e^x)}_{N}y' = 0$$

$$\frac{\partial M}{\partial y} = 2xcosy + 3y^2e^x = \frac{\partial N}{\partial x} \quad \text{EXACT}$$

$$\Longrightarrow \exists\phi \text{ such that}$$

$$\frac{\partial \phi}{\partial x} = 2xsiny + y^3e^x..........(1)$$

and

$$\frac{\partial \phi}{\partial y} = x^2cosy + 3y^2e^x............(2)$$

Note: Then the DE will become $\frac{d\phi}{dx} = 0 \Longrightarrow$ Solution $\phi = C$

$$(1) \Longrightarrow \phi = x^2siny + y^3e^x + h(y)$$

$$\Longrightarrow \frac{\partial \phi}{\partial y} = x^2cosy + 3y^2e^x + \frac{dh(y)}{dy}$$

$$(2) \Longrightarrow \frac{dh}{dy} = 0 \Longrightarrow h = 0$$

$$\Longrightarrow \phi = x^2siny + y^3e^x$$

$$\text{General solution: } x^2siny + y^3e^x = C$$

where C is an arbitrary constant.

Q2)

$$\underbrace{(3x^2 + 4xy)}_{M} + \underbrace{(2x^2 + 2y)}_{N}y' = 0$$

$$\frac{\partial M}{\partial y} = 4x = \frac{\partial N}{\partial x} \quad \text{EXACT}$$

$$\Longrightarrow \exists\phi \text{ such that}$$

$$\frac{\partial \phi}{\partial x} = 3x^2 + 4xy..........(1)$$

and

$$\frac{\partial \phi}{\partial y} = 2x^2 + 2y............(2)$$

$$(1) \implies \phi = x^3 + 2x^2y + h(y)$$

$$\implies \frac{\partial \phi}{\partial y} = 2x^2 + \frac{dh(y)}{dy}$$

$$(2) \implies \frac{dh}{dy} = 2y \implies h = y^2$$

$$\implies \phi = x^3 + 2x^2y + y^2$$

$$\text{General solution: } x^3 + 2x^2y + y^2 = C$$

where C is an arbitrary constant.

Q3)

$$\underbrace{(2xcosy + 3x^2y)}_{M} + \underbrace{(x^3 - x^2siny - y)}_{N}y' = 0; \;\; y(0) = 2$$

$$\frac{\partial M}{\partial y} = -2xsiny + 3x^2 = \frac{\partial N}{\partial x} \quad \text{EXACT}$$

$$\implies \exists \phi \text{ such that}$$

$$\frac{\partial \phi}{\partial x} = 2xcosy + 3x^2y..........(1)$$

and

$$\frac{\partial \phi}{\partial y} = x^3 - x^2siny - y...........(2)$$

$$(1) \implies \phi = x^2cosy + x^3y + h(y)$$

$$\implies \frac{\partial \phi}{\partial y} = -x^2siny + x^3 + \frac{dh(y)}{dy}$$

$$(2) \implies \frac{dh}{dy} = -y \implies h = -\frac{y^2}{2}$$

$$\implies \phi = x^2cosy + x^3y - \frac{y^2}{2}$$

$$\text{General solution: } x^2cosy + x^3y - \frac{y^2}{2} = C$$

$$y(0) = 2 \implies C = -2$$

$$\text{Solution: } \boxed{x^2cosy + x^3y - \frac{y^2}{2} = -2}$$

Q4)

$$\underbrace{(2xy-3)}_{M}+\underbrace{(x^2+4y)}_{N}y'=0;\quad y(1)=2$$

$$\frac{\partial M}{\partial y}=2x=\frac{\partial N}{\partial x}\quad \text{EXACT}$$

$$\implies \exists\phi \text{ such that}$$

$$\frac{\partial \phi}{\partial x}=2xy-3..........(1)$$

and

$$\frac{\partial \phi}{\partial y}=x^2+4y............(2)$$

$$(1)\implies \phi=x^2y-3x+h(y)$$

$$\implies \frac{\partial \phi}{\partial y}=x^2+\frac{dh(y)}{dy}$$

$$(2)\implies \frac{dh}{dy}=4y\implies h=2y^2$$

$$\implies \phi=x^2y-3x+2y^2$$

$$\text{General solution: } x^2y-3x+2y^2=C$$

$$y(1)=2\implies C=7$$

$$\text{Solution: } \boxed{x^2y-3x+2y^2=7}$$

Q5)

$$\underbrace{(ysec^2x+secxtanx)}_{M}+\underbrace{(tanx+2y)}_{N}y'=0$$

$$\frac{\partial M}{\partial y}=sec^2x=\frac{\partial N}{\partial x}\quad \text{EXACT}$$

$$\implies \exists\phi \text{ such that}$$

$$\frac{\partial \phi}{\partial x}=ysec^2x+secxtanx..........(1)$$

and

$$\frac{\partial \phi}{\partial y}=tanx+2y............(2)$$

$$(2)\implies \phi=(tanx)y+y^2+h(x)$$

$$\implies \frac{\partial \phi}{\partial x}=(sec^2x)y+\frac{dh(x)}{dx}$$

$$(1) \Longrightarrow \frac{dh}{dx} = secxtanx \Longrightarrow h(x) = secx$$

$$\Longrightarrow \phi = (tanx)y + y^2 + secx$$

$$\text{General solution: } (tanx)y + y^2 + secx = C$$

where C is an arbitrary constant.