H.W. 4 Solution

Q1)

$$y' = \frac{3y^2 - x^2}{2xy};\ x \neq 0$$

$$= \frac{3(\frac{y}{x})^2 - 1}{2(\frac{y}{x})} \quad \text{Hom. D.E.}$$

$$v = \frac{y}{x} \Longleftrightarrow y = vx \Longrightarrow y' = v + xv'$$

$$\overset{DE}{\Longrightarrow} v + xv' = \frac{3v^2 - 1}{2v}$$

$$\Longrightarrow xv' = \frac{3v^2 - 1}{2v} - v = \frac{v^2 - 1}{2v} \text{ SEP. D.E.}$$

$$\Longrightarrow \frac{2v}{v^2 - 1} v' = \frac{1}{x}$$

$$\Longrightarrow \int \frac{2v}{v^2 - 1} v' dx = ln|x| + C$$

$$\text{i.e.} \int \frac{2v}{v^2 - 1} dv = ln|x| + C$$

$$\Longrightarrow ln|v^2 - 1| = ln|x| + C$$

$$\Longrightarrow ln|(\frac{y}{x})^2 - 1| = ln|x| + C$$

Q2)

$$y + \sqrt{x^2 + y^2} - xy' = 0;\ y(1) = 0. \text{ We will solve for } \boxed{x > 0}$$

$$y' = \frac{y + \sqrt{x^2 + y^2}}{x} = \frac{y}{x} + \sqrt{1 + (\frac{y}{x})^2} \quad \text{Hom. D.E.}$$

$$v = \frac{y}{x} \Longleftrightarrow y = vx \Longrightarrow y' = v + xv'$$

$$\overset{DE}{\Longrightarrow} v + xv' = v + \sqrt{1 + v^2}$$

$$\Longrightarrow xv' = \sqrt{1 + v^2} \quad \text{SEP. D.E.}$$

$$\Longrightarrow \frac{1}{\sqrt{1 + v^2}} v' = \frac{1}{x}$$

$$\Longrightarrow \int \frac{1}{\sqrt{1 + v^2}} v' dx = lnx + C$$

$$\text{i.e.} \int \frac{1}{\sqrt{1 + v^2}} dv = lnx + C$$

$$\Longrightarrow ln|v + \sqrt{1 + v^2}| = lnx + C$$

$$y(1) = 0 \Longrightarrow C = 0$$

$$\Longrightarrow ln|(\frac{y}{x}) + \sqrt{1 + (\frac{y}{x})^2}| = lnx$$

Q3)

$$xtan\frac{y}{x} + y - xy' = 0; \ x \neq 0$$

$$y' = tan(\frac{y}{x}) + (\frac{y}{x}) \quad \text{Hom. D.E.}$$

$$v = \frac{y}{x} \Longleftrightarrow y = vx \Longrightarrow y' = v + xv'$$

$$\overset{DE}{\Longrightarrow} v + xv' = tanv + v$$

$$\Longrightarrow \frac{1}{tanv}v' = \frac{1}{x} \ \ \text{SEP. D.E.}$$

$$\Longrightarrow \int \frac{cosv}{sinv}v'dx = ln|x| + C$$

$$\text{i.e.} \int \frac{cosv}{sinv}dv = ln|x| + C$$

$$\Longrightarrow ln|sinv| = ln|x| + C$$

$$\Longrightarrow ln|sin(\frac{y}{x})| = ln|x| + C$$

Q4)

$$(\sqrt{x+y} + \sqrt{x-y}) + (\sqrt{x-y} - \sqrt{x+y})y' = 0; \ \boxed{x > 0}$$

$$y' = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}}$$

$$= \frac{\sqrt{1 + (\frac{y}{x})} + \sqrt{1 - (\frac{y}{x})}}{\sqrt{1 + (\frac{y}{x})} - \sqrt{1 - (\frac{y}{x})}} \quad \text{Hom. D.E.}$$

$$v = \frac{y}{x} \Longleftrightarrow y = vx \Longrightarrow y' = v + xv'$$

$$\overset{DE}{\Longrightarrow} v + xv' = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$\Longrightarrow xv' = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} - v$$

$$= \frac{(1+v) + 2\sqrt{1-v^2} + (1-v)}{(1+v) - (1-v)} - v$$

$$= \frac{1 + \sqrt{1-v^2}}{v} - v$$

$$= \frac{(1-v^2)+\sqrt{1-v^2}}{v} \quad \text{SEP. D.E.}$$

$$\frac{v}{(1-v^2)+\sqrt{1-v^2}}v' = \frac{1}{x}$$

$$\Longrightarrow \int \frac{v}{(1-v^2)+\sqrt{1-v^2}}v'dx = lnx + C$$

$$\text{i.e.} \quad \int (\frac{1}{\sqrt{1-v^2}+1})\frac{v}{\sqrt{1-v^2}}dv = lnx + C$$

$$\text{Let } u = \sqrt{1-v^2} \Longrightarrow \frac{du}{dv} = \frac{1}{2}\frac{(-2v)}{\sqrt{1-v^2}} = -\frac{v}{\sqrt{1-v^2}}$$

$$\int \frac{1}{u+1}(-\frac{du}{dv})dv = lnx + C$$

$$\Longrightarrow -\int \frac{1}{u+1}du = lnx + C$$

$$\Longrightarrow -ln|u+1| = lnx + C$$

$$\Longrightarrow -ln[\sqrt{1-v^2}+1] = lnx + C$$

$$\Longrightarrow -ln[\sqrt{1-(y/x)^2}+1] = lnx + C$$

Q5)

$$x^3 + y^2\sqrt{x^2+y^2} - xy\sqrt{x^2+y^2}y' = 0; \ \boxed{x > 0}$$

$$y' = \frac{x^3 + y^2\sqrt{x^2+y^2}}{xy\sqrt{x^2+y^2}}$$

$$= \frac{1+(\frac{y}{x})^2\sqrt{1+(\frac{y}{x})^2}}{(\frac{y}{x})\sqrt{1+(\frac{y}{x})^2}} \quad \text{Hom. D.E.}$$

$$v = \frac{y}{x} \Longleftrightarrow y = vx \Longrightarrow y' = v + xv'$$

$$\overset{DE}{\Longrightarrow} v + xv' = \frac{1+v^2\sqrt{1+v^2}}{v\sqrt{1+v^2}}$$

$$\Longrightarrow xv' = \frac{1+v^2\sqrt{1+v^2}}{v\sqrt{1+v^2}} - v$$

$$= \frac{1}{v\sqrt{1+v^2}} \quad \text{SEP. D.E.}$$

$$\Longrightarrow \int v\sqrt{1+v^2}v'dx = \int \frac{1}{x}dx + C$$

$$\Longrightarrow \int v\sqrt{1+v^2}dv = lnx + C$$

$$\Longrightarrow \frac{1}{2}\int (2v)\sqrt{1+v^2}dv = lnx + C$$

$$\text{Let } u = 1+v^2 \Longrightarrow \frac{du}{dv} = 2v$$

$$\Longrightarrow \frac{1}{2}\int \sqrt{u}\frac{du}{dv}dv = lnx + C$$

$$\Longrightarrow \frac{1}{2}\int \sqrt{u}du = lnx + C$$

$$\Longrightarrow \frac{1}{2}\frac{u^{3/2}}{(3/2)} = lnx + C$$

$$\Longrightarrow \frac{1}{3}[1+(\frac{y}{x})^2]^{3/2} = lnx + C$$

Q6)

$$(3x^2+9xy+5y^2)-(6x^2+4xy)y' = 0;\ y(2) = -6. \text{ We will solve for } \boxed{x>0}$$

$$y' = \frac{3x^2+9xy+5y^2}{6x^2+4xy}$$

$$= \frac{3+9(\frac{y}{x})+5(\frac{y}{x})^2}{6+4(\frac{y}{x})} \quad \text{Hom. D.E.}$$

$$v = \frac{y}{x} \Longleftrightarrow y = vx \Longrightarrow y' = v + xv'$$

$$\overset{DE}{\Longrightarrow} v + xv' = \frac{3+9v+5v^2}{6+4v}$$

$$\Longrightarrow xv' = \frac{3+9v+5v^2}{6+4v} - v$$

$$= \frac{3+3v+v^2}{6+4v} \quad \text{SEP. D.E.}$$

$$\Longrightarrow \int \frac{6+4v}{3+3v+v^2}v'dx = \int \frac{1}{x}dx$$

$$\Longrightarrow \int \frac{6+4v}{3+3v+v^2}dv = lnx + C$$

$$\Longrightarrow 2\int \frac{3+2v}{3+3v+v^2}dv = lnx + C$$

$$\Longrightarrow 2ln|3+3v+v^2| = lnx + C$$

$$\Longrightarrow 2ln|3+3(\frac{y}{x})+(\frac{y}{x})^2| = lnx + C$$

$$y(2) = -6 \Longrightarrow C = ln(\frac{9}{2})$$

$$\Longrightarrow 2ln|3+3(\frac{y}{x})+(\frac{y}{x})^2| = lnx + ln(\frac{9}{2})$$