H.W. 1 Solution

Q1)

$$y' = e^{-y}(x + x^3)$$

$$e^y y' = x + x^3$$

$$\int e^y y' dx = \int (x + x^3) dx + C$$

$$\int e^y dy = \frac{x^2}{2} + \frac{x^4}{4} + C$$

$$e^y = \frac{x^2}{2} + \frac{x^4}{4} + C$$

where C is an arbitrary constant.

Q2)

$$yy' + (1 + y^2)sinx = 0;\ y(0) = 1$$

$$y' = -\frac{(1 + y^2)sinx}{y}$$

$$\frac{y}{1 + y^2} y' = -sinx$$

$$\int \frac{y}{1 + y^2} y' dx = \int (-sinx) dx + C$$

$$\int \frac{y}{1 + y^2} dy = cosx + C$$

$$\frac{1}{2} ln(1 + y^2) = cosx + C$$

$$y(0) = 1 \Longrightarrow \frac{1}{2} ln2 = 1 + C$$

$$\Longrightarrow C = \frac{1}{2} ln2 - 1$$

$$\frac{1}{2} ln(1 + y^2) = cosx + \frac{1}{2} ln2 - 1$$

Q3)

$$(1 + y^2) + (1 + x^2) y' = 0;\ y(0) = -1$$

$$y' = -\frac{(1 + y^2)}{(1 + x^2)}$$

$$\frac{1}{1 + y^2} y' = -\frac{1}{1 + x^2}$$

$$\int \frac{1}{1+y^2} y' dx = \int -\frac{1}{1+x^2} dx + C$$

$$\int \frac{1}{1+y^2} dy = -tan^{-1}x + C$$

$$tan^{-1}y = -tan^{-1}x + C$$

$$y(0) = -1 \Longrightarrow$$

$$-\frac{\pi}{4} = 0 + C$$

$$\Longrightarrow C = -\frac{\pi}{4}$$

$$tan^{-1}y = -tan^{-1}x - \frac{\pi}{4}$$

Q4)

$$2x(y+1) - yy' = 0; \; y(0) = -2$$

$$y' = \frac{2x(y+1)}{y}$$

$$\frac{y}{y+1} y' = 2x$$

$$\int \frac{y}{y+1} y' dx = \int 2x dx + C$$

$$\int \frac{y}{y+1} dy = x^2 + C$$

$$\int \frac{(y+1)-1}{y+1} dy = x^2 + C$$

$$\int [1 - \frac{1}{y+1}] dy = x^2 + C$$

$$y - ln|y+1| = x^2 + C$$

$$y(0) = -2 \Longrightarrow -2 - 0 = 0 + C$$

$$\Longrightarrow C = -2$$

$$y - ln|y+1| = x^2 - 2$$

Q5)

$$y' = e^{y+x+3} = e^y e^{x+3}$$

$$e^{-y} y' = e^{x+3}$$

$$\int e^{-y} y' dx = \int e^{x+3} dx + C$$

$$\int e^{-y} dy = e^{x+3} + C$$

$$-e^{-y} = e^{x+3} + C$$

where C is an arbitrary constant.

Q6)

$$x siny + (x^2+1) cosy.y' = 0; \; y(1) = \frac{\pi}{2}$$

$$y' = -\frac{x siny}{(x^2+1) cosy}$$

$$\frac{cosy}{siny} y' = -\frac{x}{x^2+1}$$

$$\int \frac{cosy}{siny} y' dx = \int -\frac{x}{x^2+1} dx + C$$

$$\int \frac{cosy}{siny} dy = -\frac{1}{2} ln[x^2+1] + C$$

$$ln|siny| = -\frac{1}{2} ln[x^2+1] + C$$

$$y(1) = \frac{\pi}{2} \Longrightarrow$$

$$0 = -\frac{1}{2} ln2 + C$$

$$C = \frac{1}{2} ln2$$

$$ln|siny| = -\frac{1}{2} ln[x^2+1] + \frac{1}{2} ln2$$

Q7)

$$xy + 2x + y + 2 + (x^2+2x) y' = 0$$

$$y' = \frac{-xy - 2x - y - 2}{x^2+2x}$$

$$= \frac{-x(y+2) - (y+2)}{x^2+x}$$

$$= \frac{(y+2)(-x-1)}{x^2+2x}$$

$$= -\frac{(y+2)(x+1)}{x^2+2x}$$

$$\frac{1}{y+2} y' = -\frac{x+1}{x^2+2x}$$

$$\int \frac{1}{y+2}y'dx = \int -\frac{x+1}{x^2+2x}dx + C$$
$$\int \frac{1}{y+2}dy = -\frac{1}{2}ln|x^2+2x| + C$$
$$ln|y+2| = -\frac{1}{2}ln|x^2+2x| + C$$

Q8)

$$cosecy + secx.y' = 0$$
$$y' = -\frac{cosecy}{secx}$$
$$siny.y' = -cosx$$
$$\int siny.y'dx = \int -cosxdx + C$$
$$\int sinydy = -sinx + C$$
$$-cosy = -sinx + C$$

where C is an arbitrary constant.

Q9)

$$2r(s^2+1) + (r^4+1)\frac{ds}{dr} = 0$$
$$\frac{ds}{dr} = -\frac{2r(s^2+1)}{r^4+1}$$
$$\frac{1}{s^2+1}\frac{ds}{dr} = -\frac{2r}{r^4+1}$$
$$\int \frac{1}{s^2+1}(\frac{ds}{dr})dr = \int -\frac{2r}{r^4+1}dr + C$$
$$\int \frac{1}{s^2+1}ds = -\int \frac{2r}{r^4+1}dr + C$$
$$tan^{-1}s = -\int \frac{2r}{r^4+1}dr + C$$

Now $\int \frac{2r}{r^4+1}dr = \int \frac{1}{v^2+1}(\frac{dv}{dr})dr$

where $v = r^2$

$$= \int \frac{1}{v^2+1}dv$$
$$= tan^{-1}v$$
$$= tan^{-1}(r^2)$$

$$\Longrightarrow tan^{-1}s = -tan^{-1}(r^2) + C$$

where C is an arbitrary constant.

Q10)

$$(x-4)y^4 - x^3(y^2-3)y' = 0$$

$$y' = \frac{(x-4)y^4}{x^3(y^2-3)}$$

$$\frac{y^2-3}{y^4}y' = \frac{x-4}{x^3}$$

$$\int \frac{y^2-3}{y^4}y'dx = \int \frac{x-4}{x^3}dx + C$$

$$\int [y^{-2} - 3y^{-4}]dy = \int [x^{-2} - 4x^{-3}]dx + C$$

$$\frac{y^{-1}}{-1} - 3\frac{y^{-3}}{-3} = \frac{x^{-1}}{-1} - 4\frac{x^{-2}}{-2} + C$$

$$-\frac{1}{y} + \frac{1}{y^3} = -\frac{1}{x} + \frac{2}{x^2} + C$$

where C is an arbitrary constant.